MATCHING PARTON SHOWER AND MATRIX ELEMENTS IN QED

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We report on a high-precision calculation of the Bhabha process in QED, of interest for precise luminosity determination of low-energy electron-positron colliders. The calculation is based on the matching of exact next-to-leading order corrections with a Parton Shower algorithm. The structure of the algorithm (implemented in an improved version of the event generator `BABAYAGA`) is illustrated, with a discussion on the resulting theoretical uncertainty, of the order of 0.1%.

Keywords: QED; Bhabha; luminosity; next-to-leading corrections; Parton Shower.

1. Introduction

The measurement of the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ at flavour factories, such as DAΦNE, VEPP-2M, BES, KEK-B, PEP-II and CLEO, is of primary importance for the precise determination of the anomalous magnetic moment of the muon, $(g-2)_\mu$, and of the running of the electromagnetic coupling $\alpha_{QED}(Q^2)$. The cross section values entering R are affected by the uncertainty on the knowledge of the machine luminosity, which is, in turn, related to the uncertainty on the theoretical knowledge of the cross section of a reference QED process, typically large angle Bhabha scattering, $\mu^+\mu^-$ and $\gamma\gamma$ production. To keep under control such an uncertainty, high-precision calculations of these QED processes and relative Monte Carlo generators, are required. Large-angle Bhabha scattering is of particular interest because of its large cross section and its clean experimental signature. To simulate the experimentally relevant distributions and calculate the cross section of the Bhabha process, KLOE and CLEO collaborations make use of the QED Parton Shower (PS) generator `BABAYAGA`, developed in Refs. [1,2] with a precision of 0.5%. At present a reduction of such a theoretical systematics is needed for several reasons. First, the experimental luminosity error quoted by KLOE is presently 0.3% [3]. Secondly, the measurement of the hadronic cross section in the $\pi^+\pi^-$ channel at VEPP-2M has achieved a total systematic error of 0.6−1% in the region of the ρ resonance [4], which requires, in turn, an assessment of the collider luminosity at the level of 0.1%. Last but not least, Charm and B-factories will reach in the near future the error of 1% in the luminosity measurement.

At the 0.1% level the non logarithmic contributions present in exact next-to-leading (NLO) perturbative calculations as well as the resummed leading logarithmic contributions taken into account in the PS approach are expected to be relevant. A matching algorithm which allows to incorporate the next-to-leading terms within the PS structure of an event generator such as BABAYAGA, without double counting at first order in α of the leading corrections already accounted for by the PS, has been developed in Ref. [5] and will be reviewed in the following. An estimate of the remaining theoretical uncertainty is also discussed.

2. Matching NLO corrections with Parton Shower

A general expression for the cross section with the emission of an arbitrary number of photons, in leading-log (LL) approximation, can be cast in the following form:

$$d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \, |\mathcal{M}_{n,LL}|^2 \, d\Phi_n \quad (1)$$

where $\Pi(Q^2, \varepsilon)$ is the Sudakov form-factor accounting for the soft-photon (up to an energy equal to ε in units of the incoming fermion energy E) and virtual emissions, ε is an infrared separator dividing soft and hard radiation and Q^2 is related to the energy scale of the process. $|\mathcal{M}_{n,LL}|^2$ is the squared amplitude in LL approximation describing the process with the emission of n hard photons, with energy larger than ε in units of E. $d\Phi_n$ is the exact phase-space element of the process (divided by the incoming flux factor), with the emission of n additional photons with respect to the Born-like final-state configuration.

According to the factorization theorems of soft and/or collinear singularities, the squared amplitudes in LL approximation can be written in a factorized form. In the following, for the sake of clarity and without loss of generality, we write photon emission formulas as if only one external fermion radiates, being the generalization to the real case straightforward when including the suited combinatorial factors. The one-photon emission squared amplitude in LL approximation can be written in factorized form as

$$|\mathcal{M}_{1,LL}|^2 = \frac{\alpha}{2\pi} \frac{1+z^2}{1-z} I(k) \, |\mathcal{M}_0|^2 \, \frac{8\pi^2}{E^2 z(1-z)} \quad (2)$$

where $1-z$ is the fraction of the fermion energy E carried by the photon, k is the photon four-momentum, $I(k)$ is a function describing the angular spectrum of the photon.

The cross section calculated in Eq. (1) has the advantage that the photonic corrections, in LL approximation, are resummed up to all orders of perturbation theory. On the other side, the weak point of the formula (1) is that its expansion at $\mathcal{O}(\alpha)$ does not coincide with an exact $\mathcal{O}(\alpha)$ (NLO) result, being its LL approximation. In fact

$$d\sigma_{LL}^{\alpha} \equiv [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \quad (3)$$

whereas an exact NLO cross section can be always cast in the form

$$d\sigma^{\alpha} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 \quad (4)$$

The coefficient C_α contains the complete virtual $\mathcal{O}(\alpha)$ and the $\mathcal{O}(\alpha)$ soft-bremsstrahlung squared matrix elements, in units of the Born squared amplitude, and $|\mathcal{M}_1|^2$ is the exact squared matrix element with the emission of one hard photon. We remark that $C_{\alpha,LL}$ has the same logarithmic structure as C_α and that $|\mathcal{M}_{1,LL}|^2$ has the same singular behaviour of $|\mathcal{M}_1|^2$.

In order to match the LL and NLO calculations, we introduce the correction factors, which are by construction infrared safe and free of collinear logarithms,

$$\begin{aligned} F_{SV} &= 1 + (C_\alpha - C_{\alpha,LL}) \, , \\ F_H &= 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2} \end{aligned} \quad (5)$$

and we notice that the exact $\mathcal{O}(\alpha)$ cross section can be expressed, up to terms of $\mathcal{O}(\alpha^2)$, in terms of its LL approximation as

$$\begin{aligned} d\sigma_\alpha &= F_{SV}(1+C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 \\ &+ F_H|\mathcal{M}_{1,LL}|^2 d\Phi_1 \end{aligned} \tag{6}$$

Driven by Eq. (6), Eq. (1) can be improved by writing the resummed cross section as

$$\begin{aligned} &d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon) \\ &\times \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) |\mathcal{M}_{n,LL}|^2\ d\Phi_n \end{aligned} \tag{7}$$

The expansion at $\mathcal{O}(\alpha)$ of Eq. (7) coincides now with the exact NLO cross section Eq. (4) and all higher order LL contributions are the same as in Eq. (1).

Eq. (7) is our master formula for the matching between the exact $\mathcal{O}(\alpha)$ calculation and the QED resummed PS cross section, according to which we also generate events. The correction factors of Eq. (5) can in principle make the differential cross section of Eq. (7) negative in some point, namely where the PS approximation is less accurate. Nevertheless, we verified that this never happens when considering typical event selection criteria for luminosity at flavour factories.

2.1. *Vacuum polarization*

Besides the photonic radiative corrections considered above, also the vacuum polarization effects must be included in the master formula (7), in order to reach the required theoretical accuracy. They are accounted for by replacing the fine structure constant $\alpha \equiv \alpha(0)$ with $\alpha(q^2) = \alpha/(1-\Delta\alpha(q^2))$, according to the algorithm described in Ref. [5]. $\Delta\alpha(q^2)$ is the fermionic contribution to the photon self-energy: the leptonic and top-quark one-loop contributions can be calculated analytically in perturbation theory, while the remaining five quarks (hadronic) contribution, $\Delta\alpha^{(5)}_{hadr}$, has to be extracted from data. To evaluate $\Delta\alpha^{(5)}_{hadr}$ we use the `HADR5N` routine [6,7].

Going beyond the Born-like approximation, the cross section corrected at $\mathcal{O}(\alpha)$ including also vacuum polarization can be written as $\sigma^{\alpha}_{VP} = \sigma_{0,VP} + \sigma^{\alpha}_{SV} + \sigma^{\alpha}_{H}$, where σ^{α}_{SV} and σ^{α}_{H} are the soft plus virtual and the hard photon $\mathcal{O}(\alpha)$ corrections of photonic origin. We can go a step further and include vacuum polarization in those terms, in order to include also part of the $\mathcal{O}(\alpha^2)$ factorizable corrections.

Furthermore, we add to the Born amplitude also the Z exchange diagrams: their effect is really tiny and negligible at low energies, but can become more important (up to 0.1%) around 10 GeV.

3. Theoretical uncertainty

Since different implementations of radiative corrections beyond exact $\mathcal{O}(\alpha)$ contributions differ by higher order effects, a hint of the missing radiative corrections which dominate the theoretical accuracy can be given by comparing the predictions of `BABAYAGA` with independent event generators, such as `BHWIDE`[8], `LABSPV`[9] and `MCGPJ`[10]. As shown in Ref. [5], the results of these comparisons are very satisfactory, with differences between `BABAYAGA` and `BHWIDE` below 0.1% on cross sections and ranging up to 1% only in the tails of some distributions where the statistics is low.

A firmer estimate of the theoretical accuracy could be given by comparison with a complete two-loop calculation, which is however not available yet. However, recently there has been important progress towards the full $\mathcal{O}(\alpha^2)$ calculation. At present, two different partial contributions have been calculated: the complete virtual two-loop photonic corrections (in the limit $Q^2 \gg m_e^2$, with $Q^2 = s,\ t,\ u$) plus real radiation in soft approximation [11] and the virtual $N_f = 1$ fermionic contribution inclusive of finite mass terms [12]. In Ref. [5] the terms in `BABAYAGA` corresponding to these two approximations of the complete $\mathcal{O}(\alpha^2)$ calculation have been extracted and compared, showing excellent agreement. The relative differences don't exceed the 0.03% level. A careful inspection of the analytical expressions of the differences shows that all logarithmic terms of infrared origin present in `BABAYAGA` have the same coefficients as in the two-loop perturbative calculations, with the exception of small terms suppressed by powers of m_e^2/s.

Other $\mathcal{O}(\alpha^2)$ contributions not considered in `BABAYAGA` (and therefore sources of theoretical uncertainty) are the light pair corrections and the soft plus virtual $\mathcal{O}(\alpha)$ corrections to the real hard radiation. The impact of the former contribution has been estimated for a sample of typical event selection and energies, with t-channel virtual LL approximated formulae[13] and real pair emission in soft approximation[14]. The impact of such corrections has been found below the 0.05% level. Exact perturbative results for the soft plus virtual corrections to real hard radiation are not available yet for the complete $s+t$ Bhabha process, which is of interest for the case of low energy e^+e^- colliders. They have been calculated separately for the t- and s-channels and their impact studied at LEP conditions. Taking into account of such experience and that `BABAYAGA` contains all the infrared enhanced terms, the size of these $\mathcal{O}(\alpha^2)$ corrections can be estimated to be smaller than 0.05%. Adding in quadrature the perturbative sources of theoretical uncertainty would give a theoretical error of the order of 0.1% for all considered event selections and energies, from 1 to 10 GeV. However this value underestimates the total error associated with `BABAYAGA` at Charm and B-factories because the routine `HADR5N`, which gives the vacuum polarisation corrections, produces large errors around the J/ψ and Υ resonances, increasing the total theoretical error of `BABAYAGA` at the 0.2% level in those energy ranges.

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